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A talk of many things...

• What is compositional verification?
• What makes it important? Difficult? Fascinating?
• Compositional Symmetry Reduction
• Parameterized Compositional Verification

“The time has come,” the Walrus said,
“To talk of many things:
Of shoes--and ships--and sealing-wax--
Of cabbages--and kings--
And why the sea is boiling hot--
And whether pigs have wings.”

from *Through the Looking-Glass and What Alice Found There*, Lewis Carroll, 1872.
Compositional Verification
What is Compositional Verification?

Key Idea: Break up a program proof into local questions of limited scope.

How can one check a routine in the sense of making sure that it is right?

In order that the man who checks may not have too difficult a task the programmer should make a number of definite assertions which can be checked individually, and from which the correctness of the whole programme easily follows.

*(Checking a Large Routine, Alan Turing, 1949)*

For sequential programs, to prove {P} S1;S2 {Q}, invent “midpoint” assertion R and establish {P} S1 {R} and {R} S2 {Q} instead.
Why Compositional Verification?

Key: Break up a program proof into local questions of limited scope.

We’ll focus on compositional verification for concurrent, shared-memory programs.

• State Explosion

A program with $N$ concurrent components can have a global state space of size $2^N$ (In theory, verification is PSPACE-hard in $N$.)

Compositional verification is one way to ameliorate this “State Explosion.”

• Loose Coupling

Large, scalable programs are expected to have “loosely coupled” components.

Compositional methods aim to exploit loose coupling.
What is a Compositional Verification Method?

Key: Break up a program proof into local questions of limited scope.

A (de)compositional verification method should have

1. A sound strategy for breaking up a proof of goal $\varphi$ for program $P = P_1 \parallel ... \parallel P_N$ into independent proofs of subgoals $\theta_i$ for each $P_i$
2. A solution for possible incompleteness in this abstraction, and
3. A mechanical method to achieve #1 and #2.

We’ll focus on program invariants.
Running Example: Dining Philosophers

Philosophers $P_0 \ldots P_{N-1}$ sit around a circular table, with forks $f_0 \ldots f_{N-1}$ between them.

1. Each philosopher cycles though states Thinking, Hungry, and Eating.
2. Initially, all philosophers are Thinking, and every fork is available.
3. A philosopher may eat only if it has picked up its left and right forks.
4. To avoid deadlock, a hungry philosopher may put down a fork it has picked up earlier (this ensures mutual exclusion but not starvation-freedom).
5. After eating, a philosopher makes both its forks available.

The goal is to show that no pair of neighboring philosophers may eat at the same time.
Dining Philosophers: Global Invariance

A global invariant is an assertion, $\theta(l_0, ..., l_{N-1}, f_0, ..., f_{N-1})$, over the global state space formed by local variables $\{l_i\}$ and shared variables $\{f_i\}$.

From an experiment with SPIN:

<table>
<thead>
<tr>
<th>Num. Proc.</th>
<th>Time (sec.)</th>
<th>Num. Reachable States</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.12</td>
<td>91808</td>
</tr>
<tr>
<td>6</td>
<td>2.4</td>
<td>902800</td>
</tr>
<tr>
<td>7</td>
<td>34</td>
<td>8874944</td>
</tr>
<tr>
<td>8</td>
<td>(killed)</td>
<td>(memory &gt; 2.5GB)</td>
</tr>
</tbody>
</table>

Compositional analysis takes less than 1 second, for over 3000 processes.
Dining Philosophers: Global Invariance in a Picture

\[ \theta (l_0, \ldots, l_5, f_0, \ldots, f_5) \]
Dining Philosophers: Compositional Invariance in a Picture

\[ \theta_0(l_0, f_5, f_0) \]

\[ \theta_5(l_5, f_4, f_5) \]

\[ \theta_4(l_4, f_3, f_4) \]

\[ \theta_3(l_3, f_2, f_3) \]

\[ \theta_2(l_2, f_1, f_2) \]

\[ \theta_1(l_1, f_0, f_1) \]

“Loosely Coupled”
Local Invariants
Compositional Invariance

A compositional invariant is a collection of per-process assertions \( \{\theta_i\} \) such that for each \( i \):

1. The assertion \( \theta_i \) is limited to the “neighborhood” of process \( P_i \): i.e., the state space induced by the local variables of this process and its adjacent shared variables.

2. \( \theta_i \) is an inductive invariant for process \( P_i \)

3. The assertion \( \theta_i \) is immune to “interference” (via shared variables) from actions of adjacent processes. I.e., \( [\theta_i \land \theta_j \land T_j \Rightarrow \theta_i'] \) is valid.

(These are just the Owicki-Gries proof rules, freed from program syntax.)

**Theorem:** For a compositional invariant \( \{\theta_i\} \), the conjunction \( (\land i: \theta_i) \) is a global inductive invariant.
Calculating a Compositional Invariant

A process $P_i$ is defined by its initial states $I_i$ and transition relation $T_i$. The conditions for compositional invariance are:

1. (Inductiveness) $[I_i \Rightarrow \theta_i]$ and $[\theta_i \land T_i \Rightarrow \theta_i']$
2. (Non-Interference) $[\theta_i \land \theta_j \land T_j \Rightarrow \theta_i']$

Collecting constraints for each $i$, we get a set of simultaneous implications $
\{[F_i(\theta) \Rightarrow \theta_i']\}$
where each $F_i$ is a monotone operator on the $\theta$’s. E.g., for the 6-node ring:

$$F_0(\theta) = I_0 \lor next_0(T_0, \theta_0) \lor next_0(T_5, \theta_0 \land \theta_5) \lor next_0(T_1, \theta_0 \land \theta_1)$$

**Theorem:** The strongest compositional invariant $\{\theta_i^*\}$ is the least simultaneous fixpoint of the operators $\{F_i\}$. Computational complexity is polynomial in $N$.  

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Calculating a Compositional Invariant: in a Picture

The fixpoint may be calculated by alternating reachability with interference-closure.

Process $P_0$

Process $P_1$

Mutual Interference

reachability
interference
reachability
interference
Calculating a Compositional Invariant-II

The conditions for compositional invariance are in Horn-clause form with unknowns \( \{\theta_i\} \):

1. (Inductiveness) \([I_i \Rightarrow \theta_i]\) and \([\theta_i \land T_i \Rightarrow \theta'_i]\)
2. (Non-Interference) \([\theta_i \land \theta_j \land T_j \Rightarrow \theta'_i]\)

A Horn-clause solver may be also used to find a solution (which need not be the strongest compositional invariant).
Incompleteness

A compositional invariant is generally weaker than the set of reachable states. Thus, some properties may be true but unprovable compositionally.

Example:

```
var lock : \{0,1\} , initially lock =1

process P(i) {
    while ( true ) {
        Thinking:
        Hungry: atomic \{if ( lock =1) then lock := 0\}
        Eating: lock := 1
    }
}
```

The strongest compositional invariant is $\theta_i = true$!
(In)completeness

This can be remedied by adding auxiliary shared variables to “couple” the local state spaces together more tightly.

Example:

```plaintext
var lock : {0,1}, initially lock = 1
var last : 0..N, initially 0                       // the last process to Eat

process P(i) {
    while (true) {
        Thinking:
        Hungry: atomic { if (lock = 1) then lock := 0; last := i }
        Eating: lock := 1
    }
}

Now the strongest compositional invariant is \( \theta_i = (Eating(i) \Leftrightarrow (lock = 0) \land (last = i)) \)
Open Questions

• What sort of auxiliary information is needed for typical proofs? Are scalable programs truly loosely coupled? I.e., is the auxiliary information simple and minimal?

• Can the right auxiliary variables be discovered automatically? [for an initial attempt, see Cohen & Namjoshi, CAV 2008]

• What hints could a programmer give to simplify such inference?

• How should one design (perhaps, synthesize) programs with easy modular proofs? Are new kinds of type systems/interface specifications needed?
Neighborhood Symmetry
Dining Philosophers: Compositional Symmetry

Which symmetries are appropriate for compositional invariants?
Neighborhood Symmetry

- Nodes $m$ and $n$ have symmetric neighborhoods if there is a bijection between the edges adjacent to the nodes.
  In the ring, the edges of node $m$ are the forks $\{f_{m-1}, f_m\}$

- **Conjecture**: if nodes $m$ and $n$ have symmetric neighborhoods and the processes placed on them, $P_m$ and $P_n$, are isomorphic, then in any compositional invariant, $\theta_m$ and $\theta_n$ are isomorphic.

  This is not quite true.

One needs a **recursive** notion of neighborhood symmetry, called “balance.”

Balance Relations

- A **local symmetry** is a triple \((m, \beta, n)\) where \(\beta\) is a bijection on the edges of \(m\) and \(n\). (E.g., for a ring, the bijection \(\beta_{m,n}\) maps \(f_{m-1} \leftrightarrow f_{n-1}\) and \(f_m \leftrightarrow f_n\))

- A **balance relation** is a set of local symmetries satisfying a bisimulation-like constraint

  If \((m, \beta, n)\) is in the relation and node \(k\) is adjacent to \(m\)
  there is a node \(l\) adjacent to \(n\) such that for some \(\gamma\)
  \((k, \gamma, l)\) is in the relation
  and \(\beta\) and \(\gamma\) agree on all common edges.

**Theorem**: If \((m, \beta, n)\) is in a balance relation and the processes \(P_m\) and \(P_n\) are isomorphic, the strongest compositional invariants \(\theta_m^*\) and \(\theta_n^*\) are also isomorphic up to \(\beta\).
Discovering Balance

• Many networks with regular structure are fully balanced (e.g., ring, mesh, torus, hypercube). Note these have limited global symmetry.

• Every global symmetry group induces a balance relation
  • Star and complete networks are fully balanced as well
  • In fact, any transitive global symmetry group induces a full balance relation

• Even irregular networks can be balanced, after a bit of neighborhood abstraction
Using Balance for Symmetry Reduction

- Consider a ring with $N$ nodes: any two nodes are balanced.
  - Hence, in the strongest compositional invariant, $\theta^*_m$ and $\theta^*_n$ are isomorphic
  - So it suffices to compute just one component, say $\theta^*_0$, and derive the others by symmetry
  - The complexity thus reduces from $\text{Poly}(N)$ to a constant!

- More generally, the computation is limited to the representatives for each balance class.
Parameterized Compositional Verification

• Scalable programs are typically parametric in the number of processes

• The parameterized model checking problem (PMCP) is to automatically validate all instances of a parametric program
  • This is undecidable in general [Apt-Kozen 1986]
  • Only a handful of decidable cases are known

• What if we weaken the requirement to the construction of a modular parametric proof? This is the parameterized compositional model checking problem (PCMCP)
  • It is also undecidable in general [Namjoshi-Trefler 2016]
  • However, many undecidable cases of PMCP become decidable for PCMCP
Parameterized Compositional Verification – Decidability

• The key to decidability is exploiting balance across arbitrary-sized configurations.

• E.g., consider Dining Philosophers on rings.
  • By balance reduction, it suffices to compute $\theta^*_0$ for a ring of size $N$
  • This computation is independent of the value of $N$
  • Hence, $\theta^*_0$ computed on a ring of size 2 produces a compositional invariant for all $N$!
  • Thus, the PCMCP is decidable for rings – note that the PMCP is undecidable for rings (quite easily, too)

• A similar argument shows the decidability of PCMCP for other constant-degree networks such as mesh and tori.

• PCMCP is also decidable for non-constant degree networks (e.g., one control and many user processes) under restrictions.
Balancing Irregular Networks with Abstraction

• Example: Dining Philosophers on an arbitrary graph.
  A philosopher must gather forks on all adjacent edges in order to eat.

• Abstract philosopher behavior by ignoring the number of edges, retaining only the predicate “all forks are acquired”. Under this abstraction, any two nodes are balanced.

• By previous arguments, the PCMCP is decidable for Dining Philosophers over arbitrary graphs.

• A similar argument applies to the PCMCP over dynamic graphs as well.
Open Questions

• What types of abstractions are most useful for compositional reasoning?

• Can the right abstractions be inferred automatically?

• How can one add auxiliary variables (i.e., ensure completeness) while preserving balance?

• Could one use the symmetry results to design (perhaps, synthesize) parametric programs with easy modular proofs?
What I haven’t talked about

• Behavior-based compositional verification rules. E.g., infer $M_1 \parallel M_2 \models \varphi$ from

$$
M_1 \parallel A_2 \models A_1 \\
M_2 \parallel A_1 \models A_2 \\
\text{and } A_1 \parallel A_2 \models \varphi
$$

• Automated learning-based algorithms to infer adequate $A_1$ and $A_2$.

• Compositional rules for liveness properties and their delicate soundness proofs.

• And anything remotely practical 😊
To Sum Up

• Modular reasoning is absolutely essential to understanding and designing complex programs.

• Fundamental, fascinating questions about modular verification remain open.

• The ideal is a program design method that uses modular assertions to ease verification.
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Background Reading List – I

• **Books and Surveys**
  • de Roever et al: Concurrency Verification: *Introduction to Compositional and Noncompositional Methods*, 2001
  • Giannakopoulou, D., Namjoshi, K.S., Pasareanu, C. Compositional Reasoning, *Handbook of Model Checking*, 2018

• **Seminal Work**
  • Lamport, L.: Proving the correctness of multiprocess programs. Trans. Softw. Eng., 1977
  • Jones, C.B.: Tentative steps toward a development method for interfering programs, TOPLAS 1983

• **Behavior-based Rules**
  • Grumberg, O., Long, D.E.: Model checking and modular verification, TOPLAS 1994
  • Alur, R., Henzinger, T.A.: Reactive modules, FMSD 1999
  • McMillan, K.L.: Circular compositional reasoning about liveness, CHARME 1999

• **Fixpoint Formulations of Compositional Invariance**
  • Cousot, P., Cousot, R.: Invariance proof methods and analysis techniques for parallel programs, Automatic Program Construction Techniques, 1984
  • Flanagan, C., Qadeer, S.: Thread-modular model checking, SPIN 2003
  • Namjoshi, K.S.: Symmetry and completeness in the analysis of parameterized systems, VMCAI 2007
Background Reading List – II

- **Neighborhood Symmetry**
  - Namjoshi K.S., Trefler, R.J.: Local symmetry and compositional verification, VMCAI 2012

- **Learning auxiliary assertions**
  - Giannakopoulou, D., Pasareanu, C.S., Barringer, H.: Component verification with automatically generated assumptions, ASE 2005
  - Gupta, A., Popeea, C., Rybalchenko, A.: Predicate abstraction and refinement for verifying multi-threaded programs. POPL 2011

- **Parameterized Compositional Verification**
  - Abdulla, P.A., Haziza, F., Holík, L.: All for the price of few, VMCAI 2013

There are many, many other excellent papers on compositional reasoning theory and practice!
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