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A talk of many things…

- What is compositional verification?
- What makes it important? Difficult? Fascinating?
- Compositional Symmetry Reduction
- Parameterized Compositional Verification

“The time has come,” the Walrus said,
“To talk of many things:
Of shoes--and ships--and sealing-wax--
Of cabbages--and kings--
And why the sea is boiling hot--
And whether pigs have wings.”

from *Through the Looking-Glass and What Alice Found There*, Lewis Carroll, 1872.

Compositional Verification

What is Compositional Verification?

Key Idea: Break up a program proof into local questions of limited scope.

> How can one check a routine in the sense of making sure that it is right?
>
> In order that the man who checks may not have too difficult a task the programmer should make a number of definite assertions which can be checked individually, and from which the correctness of the whole programme easily follows.

(*Checking a Large Routine*, Alan Turing, 1949)

For sequential programs, to prove {P} S1;S2 {Q}, invent "midpoint" assertion R and establish {P} S1 {R} and {R} S2 {Q} instead.

Why Compositional Verification?

Key: Break up a program proof into local questions of limited scope.

We'll focus on compositional verification for concurrent, shared-memory programs.

- State Explosion

  A program with N concurrent components can have a global state space of size 2^N
  (In theory, verification is PSPACE-hard in N.)

  Compositional verification is one way to ameliorate this "State Explosion."

- Loose Coupling

  Large, scalable programs are expected to have "loosely coupled" components.

  Compositional methods aim to exploit loose coupling.

What is a Compositional Verification Method?

Key: Break up a program proof into local questions of limited scope.

A (de)compositional verification method should have

1. A sound strategy for breaking up a proof of goal φ for program $P = P_1 \mid\mid \ldots \mid\mid P_N$ into independent proofs of subgoals θ_i for each P_i
2. A solution for possible incompleteness in this abstraction, and
3. A mechanical method to achieve #1 and #2.

We'll focus on program invariants.

Running Example: Dining Philosophers

Philosophers $P_0 \ldots P_{N-1}$ sit around a circular table, with forks f_0 … f_{N-1} between them.

1. Each philosopher cycles though states Thinking, Hungry, and Eating.
2. Initially, all philosophers are Thinking, and every fork is available.
3. A philosopher may eat only if it has picked up its left and right forks.
4. To avoid deadlock, a hungry philosopher may put down a fork it has picked up earlier (this ensures mutual exclusion but not starvation-freedom).
5. After eating, a philosopher makes both its forks available.

The goal is to show that no pair of neighboring philosophers may eat at the same time.

Dining Philosophers: Global Invariance

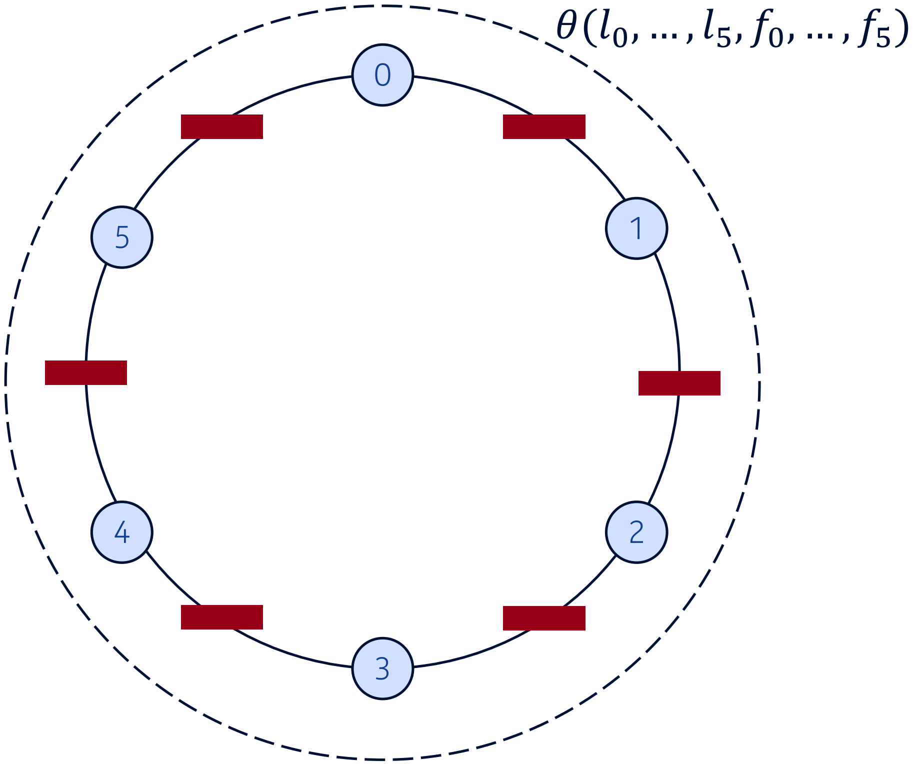
A global invariant is an assertion, $\theta(l_0, \ldots, l_{N-1}, f_0, \ldots, f_{N-1})$, over the global state space formed by local variables $\{l_i\}$ and shared variables $\{f_i\}$.

From an experiment with SPIN:

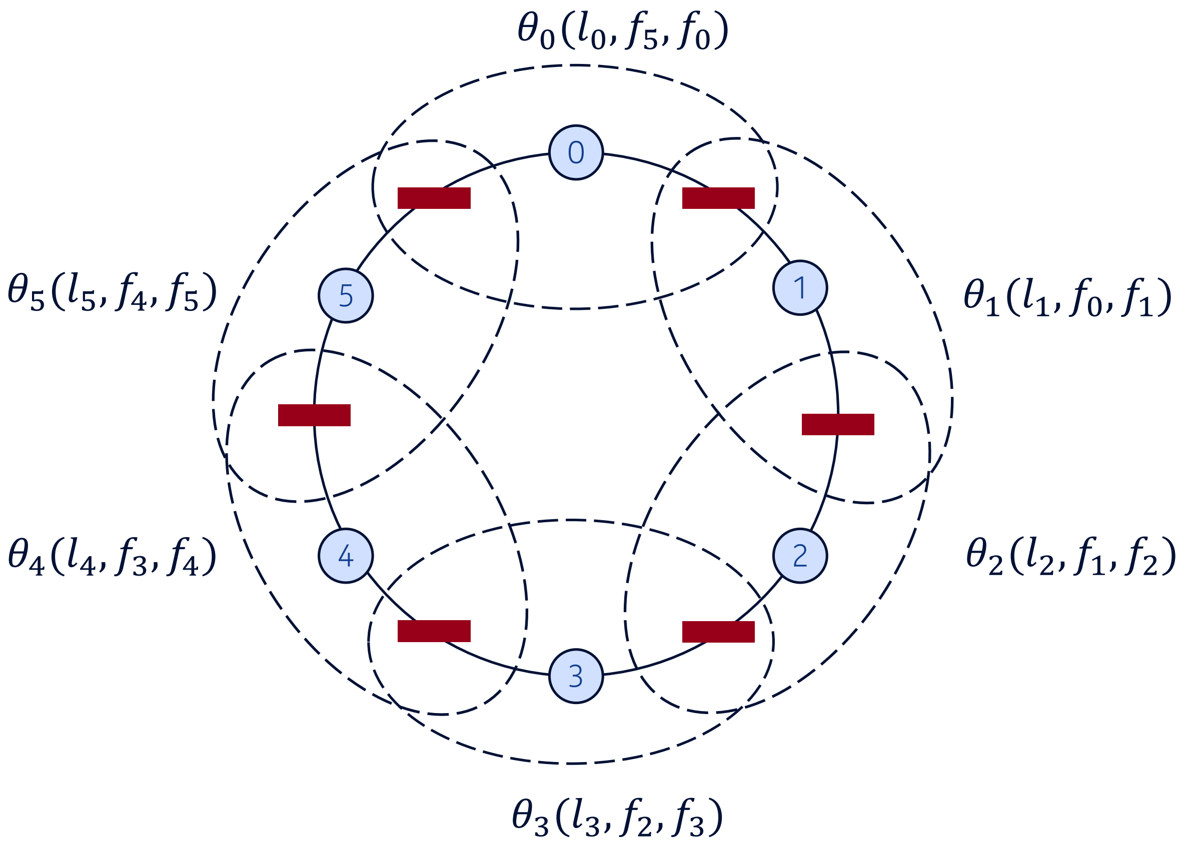
Num. Proc.	Time (sec.)	Num. Reachable States
5	0.12	91808
6	2.4	902800
7	34	8874944
8	(killed)	(memory > 2.5GB)

Compositional analysis takes less than 1 second, for over 3000 processes.

Dining Philosophers: Global Invariance in a Picture



Dining Philosophers: Compositional Invariance in a Picture



“Loosely Coupled”
Local Invariants

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Compositional Invariance

A compositional invariant is a collection of per-process assertions $\{\theta_i\}$ such that for each i:

1. The assertion θ_i is limited to the "neighborhood" of process P_i: i.e., the state space induced by the local variables of this process and its adjacent shared variables.
2. θ_i is an inductive invariant for process P_i
3. The assertion θ_i is immune to "interference" (via shared variables) from actions of adjacent processes. I.e., $[\theta_i \wedge \theta_j \wedge T_j \Rightarrow \theta_i']$ is valid.

(These are just the Owicki-Gries proof rules, freed from program syntax.)

Theorem: For a compositional invariant $\{\theta_i\}$, the conjunction $(\wedge\, i\colon \theta_i)$ is a global inductive invariant.

Calculating a Compositional Invariant

A process P_i is defined by its initial states I_i and transition relation T_i .

The conditions for compositional invariance are:

1. (Inductiveness) $[I_i \Rightarrow \theta_i]$ and $[\theta_i \wedge T_i \Rightarrow \theta_i']$
2. (Non-Interference) $[\theta_i \wedge \theta_j \wedge T_j \Rightarrow \theta_i']$

Collecting constraints for each i, we get a set of simultaneous implications

$$\{[F_i(\theta) \Rightarrow \theta_i']\}$$

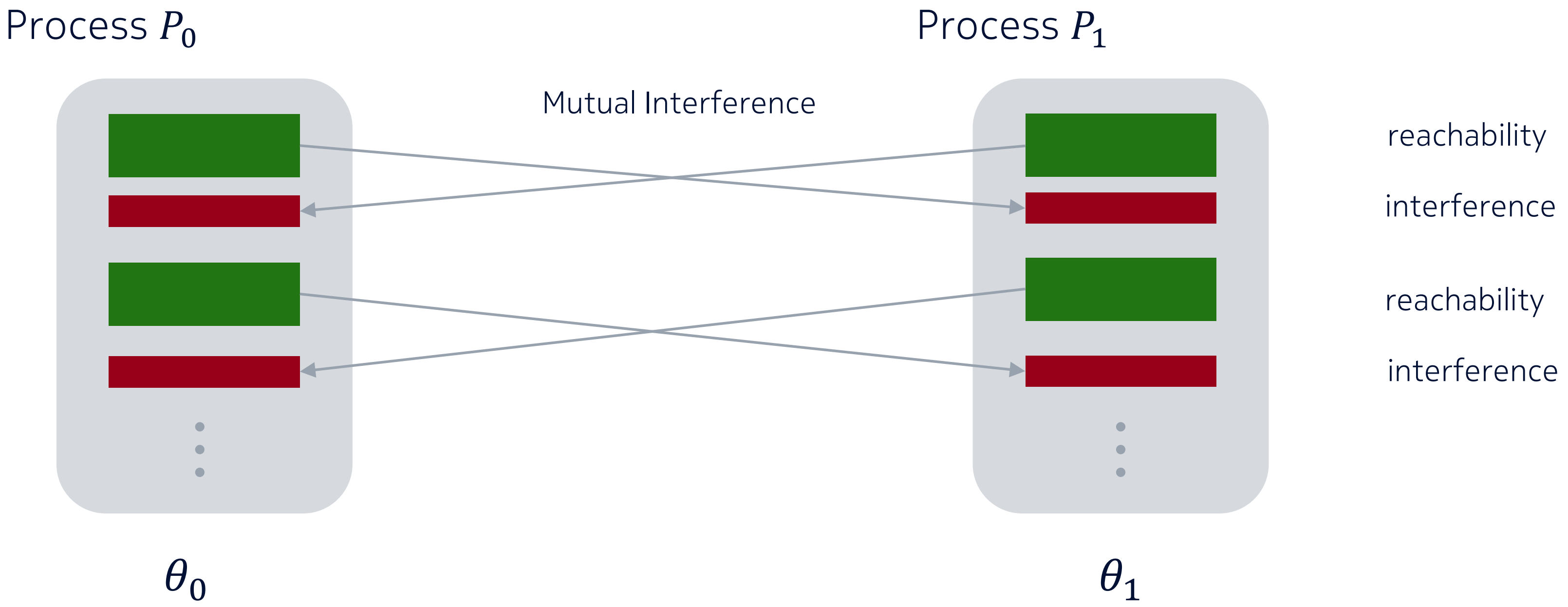
where each F_i is a monotone operator on the θ's. E.g., for the 6-node ring:

$$F_0(\theta) = I_0 \vee next_0(T_0, \theta_0) \vee next_0(T_5, \theta_0 \wedge \theta_5) \vee next_0(T_1, \theta_0 \wedge \theta_1)$$

Theorem: The strongest compositional invariant $\{\theta_i^*\}$ is the least simultaneous fixpoint of the operators $\{F_i\}$. Computational complexity is **polynomial** in N.

Calculating a Compositional Invariant: in a Picture

The fixpoint may be calculated by alternating reachability with interference-closure.



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Calculating a Compositional Invariant-II

The conditions for compositional invariance are in Horn-clause form with unknowns $\{\theta_i\}$:

1. (Inductiveness) $[I_i \Rightarrow \theta_i]$ and $[\theta_i \wedge T_i \Rightarrow \theta_i']$
2. (Non-Interference) $[\theta_i \wedge \theta_j \wedge T_j \Rightarrow \theta_i']$

A Horn-clause solver may be also used to find a solution
(which need not be the strongest compositional invariant).

Incompleteness

A compositional invariant is generally weaker than the set of reachable states. Thus, some properties may be true but unprovable compositionally.

Example:

```
var lock : {0 ,1} , initially lock =1

process P(i) {
   while ( true ) {
       Thinking:
       Hungry: atomic {if ( lock =1) then lock := 0}
       Eating: lock := 1
   }
}
```

The strongest compositional invariant is $\theta_i = true$!

(In)completeness

This can be remedied by adding **auxiliary** shared variables to “couple” the local state spaces together more tightly.

Example:

```
var lock : {0 ,1} , initially lock =1
var last : 0..N, initially 0                    // the last process to Eat

process P(i) {
   while ( true ) {
       Thinking:
       Hungry: atomic { if ( lock =1) then lock := 0; last := i }
       Eating: lock := 1
   }
}
```

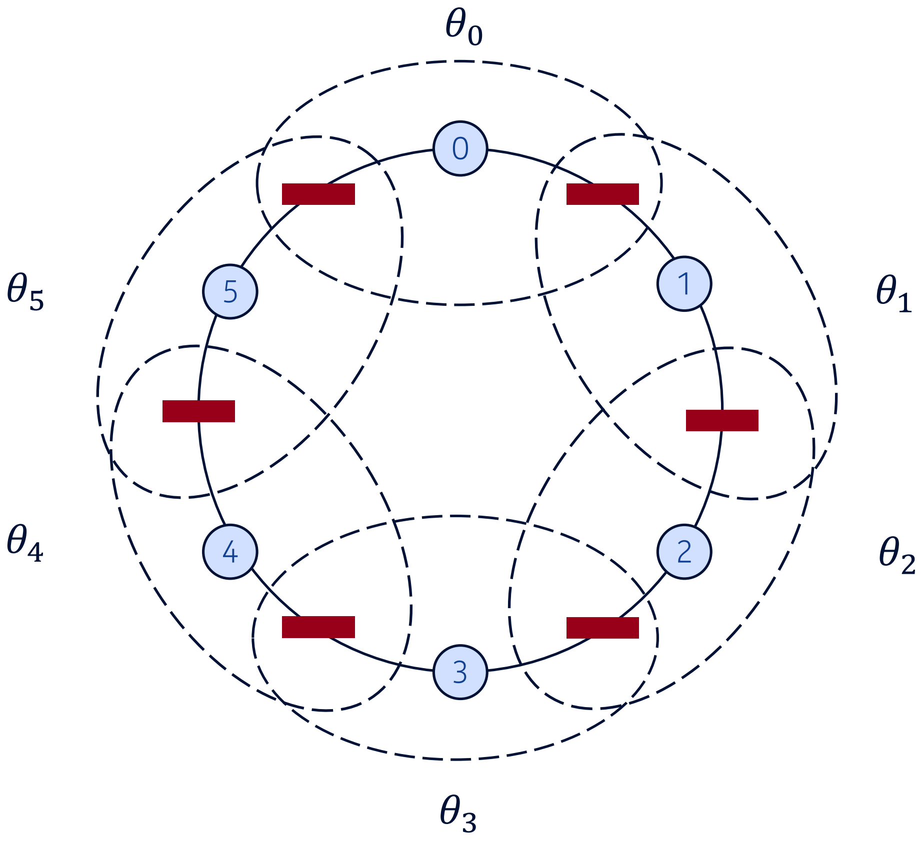
Now the strongest compositional invariant is $\theta_i = (Eating(i) \Leftrightarrow (lock = 0) \wedge (last = i))$

Open Questions

- What sort of auxiliary information is needed for typical proofs? Are scalable programs truly loosely coupled? I.e., is the auxiliary information simple and minimal?
- Can the right auxiliary variables be discovered automatically?
  [for an initial attempt, see Cohen & Namjoshi, CAV 2008]
- What hints could a programmer give to simplify such inference?
- How should one design (perhaps, synthesize) programs with easy modular proofs? Are new kinds of type systems/interface specifications needed?

Neighborhood Symmetry

Dining Philosophers: Compositional Symmetry



Which symmetries are appropriate for compositional invariants?

Neighborhood Symmetry

- Nodes m and n have symmetric neighborhoods if there is a bijection between the edges adjacent to the nodes.

  In the ring, the edges of node m are the forks $\{f_{m-1}, f_m\}$

- **Conjecture**: if nodes m and n have symmetric neighborhoods and the processes placed on them, P_m and P_n, are isomorphic, then in any compositional invariant, θ_m and θ_n are isomorphic.

  This is not quite true.

  One needs a **recursive** notion of neighborhood symmetry, called "balance."

  (cf. Martin Golubitsky and Ian Stewart: *Nonlinear dynamics of networks: the groupoid formalism*, Bull. AMS, 2006.)

Balance Relations

- A **local symmetry** is a triple (m, β, n) where β is a bijection on the edges of m and n. (E.g., for a ring, the bijection $\beta_{\{m,n\}}$ maps $f_{m-1} \mapsto f_{n-1}$ and $f_m \mapsto f_n$)

- A **balance relation** is a set of local symmetries satisfying a bisimulation-like constraint

If (m, β, n) is in the relation and node k is adjacent to m
there is a node l adjacent to n such that for some γ
(k, γ, l) is in the relation
and β and γ agree on all common edges.

Theorem: If (m, β, n) is in a balance relation and the processes P_m and P_n are isomorphic, the <u>strongest</u> compositional invariants θ_m^* and θ_n^* are also isomorphic up to β.

Discovering Balance

- Many networks with regular structure are fully balanced (e.g., ring, mesh, torus, hypercube). Note these have limited global symmetry.

- Every global symmetry group induces a balance relation
  - Star and complete networks are fully balanced as well
  - In fact, any transitive global symmetry group induces a full balance relation

- Even irregular networks can be balanced, after a bit of neighborhood abstraction

Using Balance for Symmetry Reduction

- Consider a ring with N nodes: any two nodes are balanced.
  - Hence, in the strongest compositional invariant, θ_m^* and θ_n^* are isomorphic
  - So it suffices to compute just <u>one</u> component, say θ_0^*, and derive the others by symmetry
  - The complexity thus reduces from $Poly(N)$ to a **constant**!

- More generally, the computation is limited to the representatives for each balance class.

Parameterized Compositional Verification

- Scalable programs are typically parametric in the number of processes
- The parameterized model checking problem (PMCP) is to **automatically** validate all instances of a parametric program
  - This is undecidable in general [Apt-Kozen 1986]
  - Only a handful of decidable cases are known
- What if we weaken the requirement to the construction of a **modular** parametric proof? This is the parameterized compositional model checking problem (PCMCP)
  - It is also undecidable in general [Namjoshi-Trefler 2016]
  - However, **many undecidable cases of PMCP become decidable for PCMCP**

Parameterized Compositional Verification – Decidability

- The key to decidability is exploiting balance across arbitrary-sized configurations.

- E.g., consider Dining Philosophers on rings.
  - By balance reduction, it suffices to compute θ_0^* for a ring of size N
  - This computation is **independent** of the value of N
  - Hence, θ_0^* computed on a ring of size 2 produces a compositional invariant for all N!
  - Thus, the PCMCP is decidable for rings – note that the PMCP is undecidable for rings (quite easily, too)

- A similar argument shows the decidability of PCMCP for other constant-degree networks such as mesh and tori.

- PCMCP is also decidable for non-constant degree networks (e.g., one control and many user processes) under restrictions.

Balancing Irregular Networks with Abstraction

- Example: Dining Philosophers on an arbitrary graph.
  A philosopher must gather forks on all adjacent edges in order to eat.
- Abstract philosopher behavior by ignoring the number of edges, retaining only the predicate “all forks are acquired”. Under this abstraction, any two nodes are balanced.
- By previous arguments, the PCMCP is decidable for Dining Philosophers over arbitrary graphs.
- A similar argument applies to the PCMCP over **dynamic** graphs as well.

Open Questions

- What types of abstractions are most useful for compositional reasoning?
- Can the right abstractions be inferred automatically?
- How can one add auxiliary variables (i.e., ensure completeness) while preserving balance?
- Could one use the symmetry results to design (perhaps, synthesize) parametric programs with easy modular proofs?

What I haven’t talked about

- Behavior-based compositional verification rules. E.g., infer $M_1 || M_2 \vDash \varphi$ from

$$M_1 || A_2 \vDash A_1$$
$$M_2 || A_1 \vDash A_2$$
and $A_1 || A_2 \vDash \varphi$

- Automated learning-based algorithms to infer adequate A_1 and A_2.

- Compositional rules for liveness properties and their delicate soundness proofs.

- And anything remotely practical ☺

To Sum Up

- Modular reasoning is absolutely essential to understanding and designing complex programs.

- Fundamental, fascinating questions about modular verification remain open.

- The ideal is a program design method that uses modular assertions to ease verification.

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Background Reading List – I

- **Books and Surveys**
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Background Reading List – II

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  - Namjoshi K.S., Trefler, R.J.: Local symmetry and compositional verification, VMCAI 2012
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There are many, many other excellent papers on compositional reasoning theory and practice!

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